Measuring Aerodynamic Efficiency of Falling Seeds

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1 Introduction

Falling seeds of some trees use a novel technique to increase their dispersal. Shaped like small wings, as they fall they begin to spin, and the lift from this spinning slows their fall, and increases the likelihood that they will land further from their parent tree.

There is a maximum limit of the power-efficiency of a wind turbine. This limit is called the Betz limit, and it says that no more than 59.3% of the available power in the air can be extracted by a turbine. Claims have been made that falling seeds have efficiencies that approach this limit [2].

This paper describes how the efficiency as wind-turbines can be measured for a falling seed. The key observation is that once the has settled into its spinning motion, it falls towards the ground with a constant velocity. This means that the net upward aerodynamic force F, are balanced by the downward forces due to gravity.

$$F = mg \tag{1}$$

where m is the mass of the seed. So measuring the mass establishes the aerodynamic force. The following sections describe how, by measuring the mass, spinning area, and falling velocity of a seed, its power coefficient can be determined.

2 The flow tube

This force F is produced by changing the momentum of the air as it flows past the seed. From the point of view of the seed, the air below the seed rushes upwards with a speed v_1, is slowed by the seed, so that above the seed, the air is still moving upwards, but more slowly (with velocity $v_2 < v_1$).

It can be shown [1] that the velocity at the seed, v, is exactly half-way between v_1 and v_2. We use a factor a to describe how much the incoming speed and outgoing speed differ from the speed at the seed.

The velocity before the seed is v_1, the velocity at the seed is $v_1(1-a)$ and the velocity after the seed is $v_1(1-2a)$. a is called the *axial induction factor*. This is shown in Figure 1.

2.1 Conservation of Mass

The quantity of air before the seed must be the same as the quantity of air after the seed. This means that the tube before must have the same volume as the tube after.

The tube is shorter and wider above the seed (because the seed slows the air down). This means that the area above A_2 must be greater than A_1.

The mass per second flowing in a tube is

$$m = \rho A v$$

where A is the area, v is the velocity and ρ is the density.

3 The force on the seed

The force on the seed is the difference in the momentum of the air before the seed (per unit time), and after the seed. In other words,

$$F = m_1 v_1 - m_2 v_2$$

Substituting in for the mass ($m = \rho A v$) we get

$$F = 2Aa\rho v_1^2 (1-a) \tag{2}$$

4 Working out a from F

We can rearrange Equation 2 to get

$$a(1-a) = \frac{F}{2A\rho v_1^2} \tag{3}$$

we can measure everything on the right-hand side of this equation. We'll call that c where

$$c = \frac{F}{2A\rho v_1^2} \tag{4}$$

We then get a quadratic equation for a

$$a^2 - a + c = 0 \tag{5}$$

and solving we get an expression for a.

$$a = \frac{1 \pm \sqrt{1-4c}}{2} \tag{6}$$

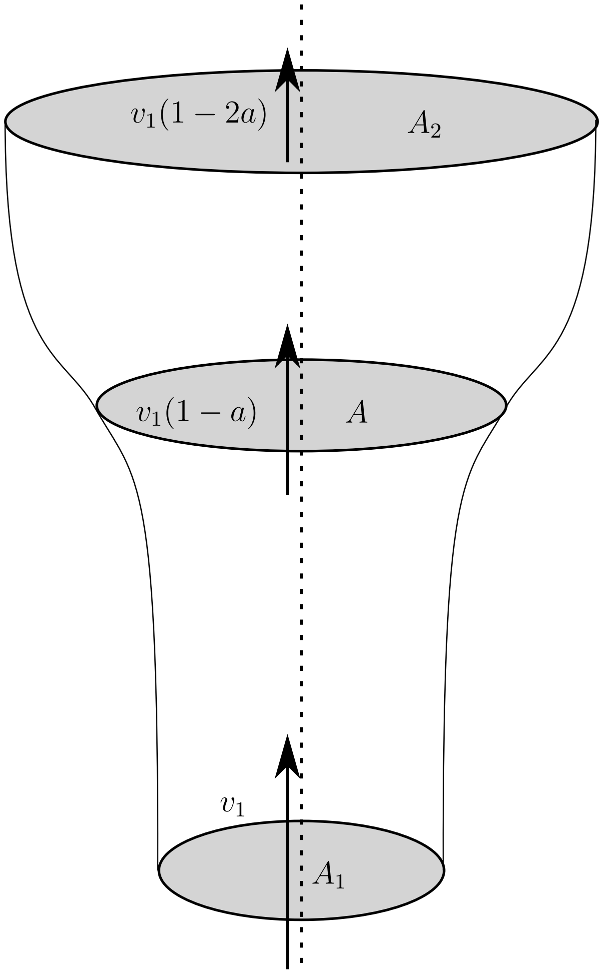

Figure 1: The flow tubes, before and after the falling seed. The wind approaches the disk of the seed with velocity v_1, and then slows down to velocity $v_1(1-2a)$ in the tube after the seed. At the seed, the velocity is half-way between $v_1(1-a)$.

or

$$a = \left[\frac{1}{2} - \frac{\sqrt{A\rho v_1^2 - 2F}}{2\sqrt{A}\sqrt{\rho}v_1}, \quad \frac{1}{2} + \frac{\sqrt{A\rho v_1^2 - 2F}}{2\sqrt{A}\sqrt{\rho}v_1} \right]$$

There are two solutions for a. However only one of them will make sense as a must be less than 0.5, or the velocity after the seed is negative!

5 The power

The power is given by the force times the velocity. This becomes:

$$P = 2Aa\rho v_1^3 (a-1)^2$$

6 The Betz Limit

The maximum possible power is the kinetic energy contained in the air flowing with velocity. This happens when all the kinetic energy of the air below the seed is used by the seed to slow it down.

$$P_{limit} = \frac{1}{2} m_1 v_1^2 = \frac{A\rho}{2} v_1^3$$

Achieving this maximum is impossible because if the air before the turbine slows down to zero speed, then it has no energy left to spin the turbine (or to do any useful work on the seed).

The coefficient C_p is the ratio of the power to the maximum possible power. It is given by:

$$C_p = \frac{P}{P_{max}} = 4a (a-1)^2$$

Betz showed that the maximum possible C_p was achieved when $a = \frac{1}{3}$, and has the value of $\frac{16}{27}$.

7 Procedure

The procedure for calculating C_p is as follows. Measure

1. mass m of the seed, and calculate the force $F = mg$
2. spinning area, A, of the seed,
3. its falling velocity v_1.
4. Look up the density of air ρ.

Using these, calculate c from Equation 4, and find both values of a from Equation 6. Choose the value for a that lies between 0 and 0.5, and from this, calculate $C_p = 4a (a-1)^2$ and compare this against the Betz Limit of $\frac{16}{27} \sim 0.593$.

References

[1] Martin OL Hansen. *Aerodynamics of wind turbines.* Routledge, 2015.

[2] Jacob R Holden, Thomas M Caley, and Mark G Turner. Maple seed performance as a wind turbine. In *53rd AIAA Aerospace Sciences Meeting*, page 1304, 2015.